

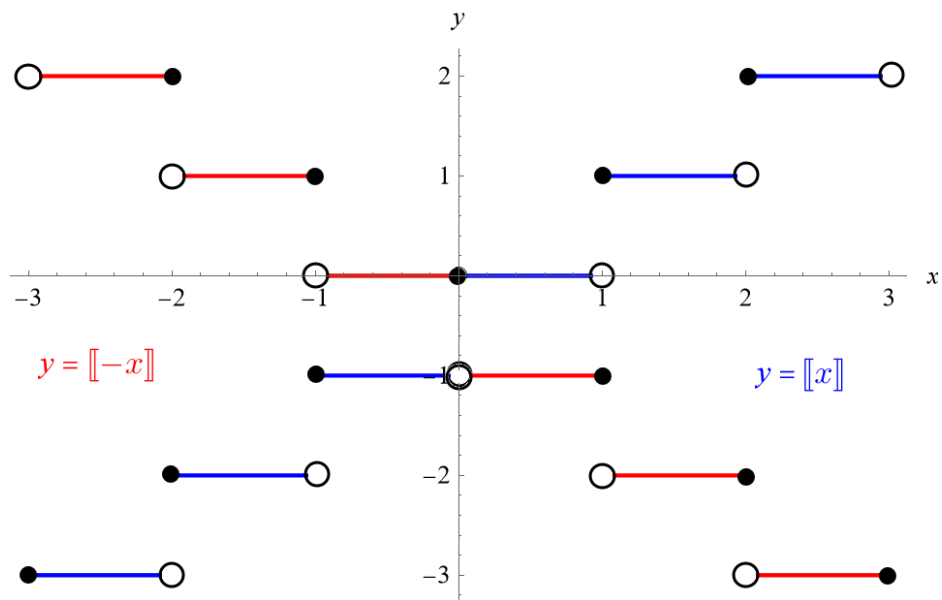
Exercise 54

Let $f(x) = \llbracket x \rrbracket + \llbracket -x \rrbracket$.

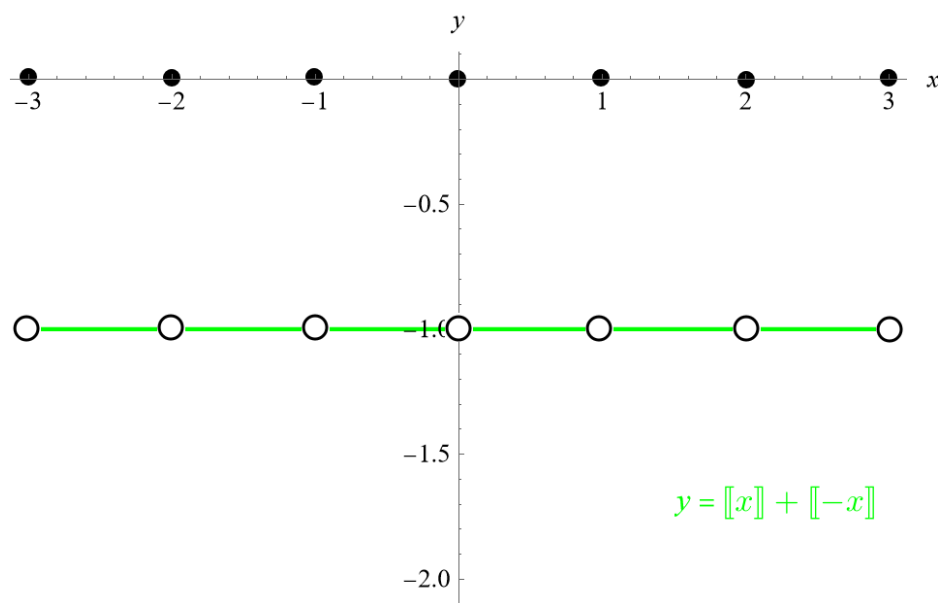
- (a) For what values of a does $\lim_{x \rightarrow a} f(x)$ exist?
- (b) At what numbers is f discontinuous?

Solution

Below are the graphs of $\llbracket x \rrbracket$ and $\llbracket -x \rrbracket$ superimposed.



With this, the graph of $f(x)$ can be drawn.



Part (a)

$\lim_{x \rightarrow a} f(x)$ exists at all values of a because the left-hand and right-hand limits are equal everywhere:

$$\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = -1.$$

Part (b)

f is discontinuous at all numbers that are integers,

$$x = n, \quad n = 0, \pm 1, \pm 2, \dots,$$

because

$$-1 = \lim_{x \rightarrow n} f(x) \neq f(n) = 0.$$